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THE EFFECT OF A FACTOR PRICE CHANGE  
ON THE EXCESS DEMAND OF THE MARKET  
AND THE NUMBER OF FIRMS

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JOHNSON COUNTY COMMUNITY COLLEGE

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OF THE MARKET AND NUMBER OF FIRMS

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Kyong-Mal Kim

The effect of a factor price change on the long-run cost curves of a perfectly competitive firm has been formerly examined.<sup>1</sup> It has been assumed in the above analysis that output price adjusts instantaneously to the new minimum average cost of the firm after a change in a factor price.<sup>2</sup> Based upon the above assumption, on which a disequilibrium situation of the market can be visualized, the effect of a factor price change on the number of the firms in the industry has also been discussed.<sup>3</sup> However, the above discussion of the effect of a factor price change on the number of the firms in a disequilibrium situation has never been formalized.<sup>4</sup> The objective of this article is to build a model, by which the effect of a factor price change on the excess demand of the market and number of the firms in a disequilibrium condition can be formerly or mathematically examined. In other words, I will construct the slopes of the firm's minimum average cost curve, which is derived from the change in a factor price, and investigate the way in which the change in a factor price affects excess demand of the market and the number of the firms in the industry in terms of the slopes of the minimum average cost of the firm.

## I. A BASIC MODEL

The following is the proposed model of the entry or exit of the firms based on the assumptions stated above.

$$dN = E_d/q = (Q_d - \bar{Q}_s)/q = [Q_d(P) - \bar{N}q(P)]/q(P) = [Q_d(C_a^*) - \bar{N}q(C_a^*)]/q(C_a^*)$$

$N$  and  $dN$  refer to the number of the firms and the change in the number of the firms respectively, and  $E_d$  and  $q$  refer to excess demand of the market and output of the firm respectively;  $Q_d$  and  $Q_s$  refer to the quantity of output demanded and supplied respectively in the market.  $P$  and  $C_a^*$  denote the prevailing price of output in the market and the minimum long-run average cost of the firm, respectively. Although  $Q_d$  and  $q$  are functions of  $P$ , their values are always measured at  $C_a^*$  in our model. The conclusion of this model is that an appearance of excess demand of the market, after a change in a factor price, will attract new firms to enter the industry, while negative excess demand of the market will force some of the existing firms out of the industry.<sup>5</sup>

## II. THE SLOPE OF THE MINIMUM AVERAGE COST CURVE

Because the concern of this section is to investigate the effect of a factor price change in terms of the slope of the firm's minimum average cost curve, we begin with the derivation of the slope of the minimum average cost curve of the firm resulting from the change in a factor price. The following model describes the long-run equilibrium of a perfectly competitive firm in a competitive industry:

$$(1) \quad q = f(x_1, x_2, \dots, x_n)$$

$$(2) \quad C = \sum_{i=1}^n p_i x_i$$

$$(3) \quad C_a = \frac{C}{q} = \sum_{i=1}^n \frac{p_i x_i}{f(x_i)}$$

where  $q$  is the rate of physical output,  $C$  is total cost and  $C_a$  is average cost, and  $x_i$  and  $p_i$  are the rate of physical factor and the price of the  $i$ th factor.

Furthermore the following assumptions on the production function are made:

$$(4) \quad f_i > 0 \quad (f_i = \frac{dq}{dx_i})$$

The second order condition based on factor efficiency as required in (7) is that:

$$(5) \quad [F] = \begin{bmatrix} 0 & f_j \\ f_i & f_{ij} \end{bmatrix} \quad (i, j = 1, 2, \dots, n)$$

be a negative definite quadratic form. And second order condition based on profit maximization as required in (7) is that:

$$(6) \quad [F^{**}] = [f_{ij}] \quad (i, j = 1, 2, \dots, n)$$

be a negative definite quadratic form. Denote their determininants by  $F$  and  $F^{**}$  and corresponding cofactor by subscripts, where the rows and columns of  $F$  are numbered 0, 1, 2, .....,  $n$  and of  $F^{**}$  are numbered 1, 2, 3, .....,  $n$ .

To derive the long-run equilibrium condition of the firm, minimize the long-run average cost by differentiating (3) while holding factor prices constant and put  $dC = 0$ . This gives<sup>6</sup>:

$$(7) \quad p_i - C_a f_i = 0 \quad (i = 1, 2, \dots, n)$$

Then differentiate (1) and (7) to derive the relevant system of equations.

$$(8) \quad \sum_{j=1}^n f_j dx_j = dq$$

$$f_i dC_a + C_a \sum_{j=1}^n f_{ij} dx_j = dp_i \quad (i = 1, 2, \dots, n)$$

Evaluate the system (8) for the case in which the first factor price changes and other input prices are constant, which leads to

$$(9) \quad \sum_{j=1}^n f_j dx_j = dq$$

$$f_1 dC_a + C_a \sum_{j=1}^n f_{1j} dx_j = dp_1$$

$$f_i dC_a + C_a \sum_{j=1}^n f_{ij} dx_j = 0 \quad (i = 2, 3, \dots, n)$$

Solving (9) by Cramer's Rule and denoting \* for equilibrium value, the equation derived is:

$$(10) \quad dC_a^*/dp_1 = C_a^*(dq^*/dp_1)(F_{00}/F) + F_{10}/F \quad (dq \neq 0)$$

Furthermore, the following results were shown from previous work.<sup>7</sup>

$$(11) \quad \sum_{j=1}^n f_j F_{ij}^{**} = -(x_1 F / \lambda C) \eta_{1c} = -F_{10}$$

$$(12) \quad dq^*/dp_1 = (x_1 F / CF^{**})(1 - \eta_{1c})$$

where  $\eta_{1c}$  refers to the expenditure elasticity of the first factor,  $1/\lambda$  refers to marginal cost and  $x_1$  refers to the first factor. Now substitute (11) and (12) in (10), where we have:

$$dC_a^*/dp_1 = (x_1/C)[C_a^* + \eta_{1c}(1/\lambda - C_a^*)] \quad (dq \neq 0)$$

noting that  $1/\lambda = C_a^*$  according to (7), the above equation becomes

$$(13) \quad dC_a^*/dp_1 = (x_1/C)C_a^* > 0 \quad (dq \neq 0)$$

Thus, when a factor price increases, the equilibrium value of the long-run average cost increases always irrespective of factor classification when the level of output is allowed to vary inasmuch as  $(x_1/C)$  and  $C_a^*$  are positive.

Finally, to construct the slope of the minimum average cost curve of the firm resulting from the change in a factor price, divide (13) by (12) where we have

$$(14) \quad dC_a^*/dq^* = (F^{**}/F)C_a^*/(1 - \eta_{1c})$$

Thus the slope of the supply curve of the firm will be positive for inferior or normal factors ( $\eta_{1c} < 1$ ), and negative for superior factors ( $\eta_{1c} > 1$ ) inasmuch as  $(F^{**}/F)$  and  $C_a^*$  are positive.

### III. EFFECT ON EXCESS DEMAND

In order to investigate the effect of the change in a factor price on excess demand of the market, the previously established model is restated below.

$$(15) \quad E_d = Q_d(C_a^*) - Nq(C_a^*)$$

Differentiating (15) while holding  $N$  constant, we have:

$$\begin{aligned} (16) \quad dE_d &= \frac{\partial Q_d(C_a^*)}{\partial C_a^*} dC_a^* - N \frac{\partial q(C_a^*)}{\partial C_a^*} dC_a^* = dC_a^* \left\{ \frac{\partial Q_d(C_a^*)}{\partial C_a^*} - N \frac{\partial q(C_a^*)}{\partial C_a^*} \right\} \\ &= dC_a^* \left\{ \frac{1}{\frac{\partial C_a^*}{\partial Q_d(C_a^*)}} - \frac{1}{\frac{1}{N} \frac{\partial C_a^*}{\partial q(C_a^*)}} \right\} = dC_a^* \left\{ \frac{1}{M_d} - \frac{1}{m_s/N} \right\} \\ &= dC_a^* \left\{ \frac{1}{M_d} - \frac{1}{M_s} \right\} \end{aligned}$$

where  $M_d$ ,  $m_s$ , and  $M_s$  refer to the slopes of the market demand curve, the firm's minimum average cost curve and the industry minimum average cost curve, respectively. Note that the slope of the firm's minimum average cost curve resulting from a factor price change is already derived in (14) and the slope of the industry minimum average cost curve can be derived by dividing the slope of the firm's minimum average cost curve by the number of the firms. The multiplication of  $1/M_d$  by  $dC_a^*$  gives the change in the quantity demanded and of  $1/M_s$  by  $dC_a^*$  gives the change in the quantity supplied. Thus, (16) indicates the difference between the change in the quantity demanded and supplied or the change in excess demand of the market for an arbitrarily given change in  $C_a^*$ .

Now in order to investigate the effect of the change in a factor price on excess demand of the market, divide (16) by  $dp_1$  where we have:

$$(17) \quad \frac{dE_d}{dp_1} = \frac{dC_a^*}{dp_1} \left\{ \frac{1}{M_d} - \frac{1}{m_s/N} \right\} = \frac{dC_a^*}{dp_1} \left\{ \frac{1}{M_d} - \frac{1}{M_s} \right\}$$

The sign of (17) will be determined by the terms in the bracket inasmuch as  $dC_a^*/dp_1$  is positive according to (13). Thus, we have:

$$(18) \quad dE_d/dp_1 \gtrless 0 \quad \text{accordingly as } 1/M_d \gtrless N/m_s (= 1/M_s) \quad (dp_1 > 0)$$

In order to make a qualitative judgement of (18) for a specific case of our particular interest where  $M_d < 0$ , substitute (14) into (18) noting that

$m_s = dC_a^*/dq^*$ , where we have:

$$(19) \quad dE_d/dp_1 \gtrless 0 \quad \text{accordingly as } \frac{1}{M_d} \gtrless \frac{(1-\eta_{1c})}{(F^{**}/F_a^*)/N} \quad (dp_1 > 0)$$

Since the value of the denominator of the right hand side of the inequality of (19) is positive, the qualitative judgment of (19) can only be made when the value of  $(1-\eta_{1c})$  is positive in which case  $dE_d/dp_1 < 0$ . Recall that  $(1-\eta_{1c})$  is the determinant of the sign of the slope of the minimum average cost curve of the firm according to (14).

#### IV. EFFECT ON NUMBER OF FIRMS

Next we investigate the effect of the change in a factor price on the number of the firms. Differentiate (15) with respect to  $p_1$  while letting  $N$  vary and put

$dE_d/dp_1 = 0$ ; a simple algebraic manipulation leads to<sup>8</sup>:

$$(20) \quad \frac{dN}{dp_1} = \left\{ \frac{\partial Q_d(C_a^*)}{\partial C_a^*} \frac{dC_a^*}{dp_1} - N \frac{\partial q(C_a^*)}{\partial C_a^*} \frac{dC_a^*}{dp_1} \right\} \frac{1}{q(C_a^*)}$$

$$= \frac{dC_a^*}{dp_1} \left\{ \frac{1}{M_d} - \frac{1}{m_s/N} \right\} \frac{1}{q(C_a^*)}$$

Before we proceed in our analysis, it is worthwhile here to test the consistency of the model. The consistency of the model after the change in a factor price requires that excess demand of the market must be accompanied by an increase in the number of the firms and negative excess demand of the market by a decrease in the number of the firms. Or stated mathematically, the requirement for the consistency of the model is  $(dN/dp_1) / (dE_d/dp_1) > 0$ . To conduct this test, substitute (17) in (20) where we have:

$$(21) \quad \frac{dN}{dp_1} = \frac{dE_d}{dp_1} \frac{1}{q(C_a^*)}$$

Rewrite (21) as

$$(22) \quad \frac{dN}{dp_1} \bigg/ \frac{dE_d}{dp_1} = \frac{dN}{dE_d} = \frac{1}{q(C_a^*)} > 0$$

Above, (22) indicates that the direction of the change in excess demand of the market and the change in the number of the firms resulting from the change in a factor price is the same. Thus, appearance of positive excess demand of the market always leads to the entry of the firms and negative excess demand of the market always leads to the exit of the firms. In other words, excess demand (negative excess demand) of the market is always a correct signal for the entry (exit) of the firms.

Now we return to the investigation of the number of the firms resulting from a factor price change. The sign of (20) will be determined by the terms in the bracket inasmuch as  $dC_a^*/dp_1$  and  $1/q(C_a^*)$  are positive. Thus we have:

$$(23) \quad dN/dp_1 \gtrless 0 \text{ accordingly as } 1/M_d \gtrless N/m_s (=1/M_s) \quad (dp_1 > 0)$$

Again in order to have a qualitative judgment of (23) for a specific case where  $M_d < 0$ , substitute (14) in (23) where we have

$$(24) \quad dN/dp_1 \gtrless 0 \text{ accordingly as } \frac{1}{M_d} \gtrless \frac{(1-\eta_{1c})}{\frac{1}{m_s} - \frac{1}{N}} \quad (dp_1 > 0)$$



The conclusions of (23) and (24) are the same as that of (18) and (19). This is an expected result since the consistency of the model is proven in (22). The qualitative judgment of (24) can only be made when the slope of the minimum average cost curve of the firm is positive or  $(1-\eta_{1c})$  is positive in which case  $dN/dp_1 < 0$ .

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- 1 See C. E. Ferguson and Thomas Saving.
- 2 This assumption has been commonly used by others. See Lowell Bassett and Thomas Borcharding, Paul Meyer, and Eugene Silberberg.
- 3 See C. E. Ferguson and Thomas Saving, Section II.
- 4 The above disequilibrium situation is a hypothetical situation in which each firm hypothesizes that output price will be pushed to the new minimum average cost at which a disequilibrium of the market can be visualized. Although, the above disequilibrium situation of the market will never be realized before the entry (exit) of the firms takes place, each firm will base its entry decision on the hypothesized or anticipated output price and disequilibrium situation of the market, as I will show later in my model.
- 5 As mentioned earlier, the above excess demand (negative excess demand) of the market is hypothetical measurement based on the anticipation that output price will be adjusted to the new minimum average cost of the firm, at which a disequilibrium of the market is visualized.
- 6 The above differentiation gives 
$$\sum_{i=1}^n p_i dx_i / \sum_{i=1}^n f_i dx_i = C_a$$
 which shows the long-run equilibrium condition in the product market since the terms on the left hand side of the equation are marginal cost. In order to derive (7) from this equation, note 
$$\sum_{i=1}^n p_i dx_i / \sum_{i=1}^n f_i dx_i = p_i / f_i$$
 as required for factor efficiency. See Jacob Mosak for this note on the factor efficiency. See Jacob Mosak for this note on the factor efficiency requirement. On the other hand, (7) shows the equilibrium condition in the factor market since it can be shown as  $p_i = Pf_i$  (or factor price is equal to the marginal revenue product).
- 7 See C. E. Ferguson and Thomas Saving, p. 777.
- 8 It is assumed here that N is non-zero and the rate of change in the number of the firms can be analyzed by means of differential equations.